International Recruitment of IPParis Schools for Engineering Cycle

Recommended knowledge in Mathematical Sciences

1 - ALGEBRA

1.1 Set theory
Operations on sets, characteristic functions. Maps, injectivity, surjectivity.
Direct and inverse image of a set.
Integer numbers, finite sets, countability.

1.2 Numbers and usual structures
Composition laws; groups, rings, fields.
Equivalence relations, quotient structures.
Real numbers, complex numbers, complex exponential.
Application to plane geometry.
Polynomials, relations between the roots and the coefficients.
Elementary arithmetics (in **Z/nZ**).

1.3 Finite dimensional vector spaces (*)
Free families, generating families, bases, dimension.
Determinant of n vectors; characterization of bases.
Matrices, operations on matrices.
Determinant of a square matrix; expansion with respect to a line or to a column; rank, cofactors.
Linear maps, matrix associated to a linear map.
Endomorphisms, trace, determinant, rank.
Linear systems of equations.

1.4 Reduction of endomorphisms
Stable subspaces.
Eigenvalues, eigenvectors of an endomorphism or a square matrix; similar matrices; geometrical interpretation.

Characteristic polynomial, Cayley-Hamilton theorem.
Reduction of endomorphisms in finite dimension; diagonalizable endomorphisms and matrices.

1.5 Euclidean spaces, Euclidean geometry

Scalar product; Cauchy-Schwarz inequality; norms and associated distances.
Euclidean spaces of finite dimension, orthonormal bases; orthogonal projections.
Orthogonal group O(E); orthogonal symmetries.
Orthogonal matrices; diagonalization of symmetric real matrices.
Properties of orthogonal endomorphisms of $\mathbf{R^2}$ and $\mathbf{R^3}$.

(*) In several countries linear algebra is studied only in $\mathbf{R}^k$ or $\mathbf{C}^k$; the candidates from these countries are strongly advised to get familiar with the formalism of abstract vector spaces.

2 - ANALYSIS AND DIFFERENTIAL GEOMETRY

2.1 Topology in finite dimensional normed vector spaces

Open and closed sets, accumulation points, interior points.
Convergent sequences in normed vector spaces; continuous mappings.
Compact spaces, images of compact sets by continuous mappings, existence of extrema.
Equivalence of norms.

2.2 Real or complex valued functions defined on an interval

Derivative at a point, functions of class C^k.
Mean value theorem, Taylor's formula.
Primitive of continuous functions.
Usual functions (exponential, logarithm, trigonometric functions, rational fractions).
Sequences and series of functions, simple and uniform convergence.

2.3 Integration on a bounded interval

Integral of piecewise continuous functions.
Fundamental theorem of calculus (expressing the integral of a function in terms of a primitive).
Integration by parts, change of variable, integrals depending on a parameter.
Continuity under the sign $\int$, differentiation under the sign $\int$.
Cauchy-Schwarz inequality.

2.4 Series of numbers, power series

Series of real or complex numbers, simple and absolute convergence.
Integral comparison criterion, product of absolutely convergence series.
Power series, radius of convergence; function that can be expanded in a power series on an interval.
Taylor series expansion of e^t, sin(t), cos(t), ln (1+t), $(1+t)^a$ where a is a real number.

2.5 Differential equations

Linear scalar equations of degree 1 or 2, fundamental systems of solutions.
Linear systems with constant coefficients.
Method of the variation of the constants.
Notions on non-linear differential equations.

2.6 Functions of several real variables

Partial derivatives, differential of a function defined on $\mathbf{R}^k$.
Chain rule.
C^1-functions; Schwarz theorem for C^2-functions.
Diffeomorphisms, inverse function theorem. Critical points, local and global extrema.
Plane curves; tangent vector at a point, metric properties of plane curves (arc length, curvature).
Surfaces in $\mathbf{R}^3$, tangent plane to a surface defined by a Cartesian equation $F(x,y,z) = 0$.